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Spontaneous Symmetry Breaking in a Time-Dependent Space-Time

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Spontaneous symmetry breaking of a ϕ^4 quantum of field theory in a time-dependent space-time, de Sitter space, is discussed in the Schrödinger picture. Instead of the usual cutoff method we use an ε-regularization procedure to deal with the divergent integrals.

1. INTRODUCTION

We discuss the spontaneous symmetry breaking (SSB) of $\lambda\varphi^4$ in $3+1$ dimensions, in a time-dependent space-time and in the Schrödinger picture (Jackiw and Kerman, 1979; Cooper *et al.*, 1986; Castorina and Consoli, 1983; Cornwall *et al.*, 1974). Our method is nonperturbative and some of the nonlinear features of the full quantum theory are retained. Recently this method has been successfully applied to $\lambda\varphi^4$ theory in both flat (Pi and Samiullah, 1987; Cooper and Mottola, 1987) and curved spaces (Roy, 1991; Kim *et al.*, 1988) and also to the Liouville model (Roy and Roy, 1991).

In a recent paper Branchina *et al.* (1990) discussed the nontriviality of spontaneously broken $\lambda\varphi^4$ theory. They argued that $\lambda\varphi^4$ theories undergoing spontaneous symmetry breaking are asymptotically free. It is also found that if the cutoff is sent to infinity with respect to the renormalized mass, the theory develops another scale which corresponds to spontaneous symmetry breaking (Consoli and Ciancitto, 1985). Their renormalization gives the same result as that of the so-called "autonomous theory" of Hajj and Stevenson (1988). Here we follow the renormalization procedure of Branchina *et al.* to study SSB in what can be considered as half the de Sitter space.

De Sitter space is the most widely studied curved space-time in quantum field theory, as it is the unique maximally symmetric curved space-time. It

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enjoys the same degree of symmetry as Minkowski space (except dilation), which facilitates computations in quantum field theory (Birell and Davies, 1982).

In the present paper, our renormalization, the ε-regularization technique, is similar to dimensional regularization. This method is simpler than the usual cutoff method (Latorre *et al.*, 1986). For an elaborate treatment of renormalization in $\lambda\varphi^4$ theory see Barnes and Ghandour (1980) and Bardeen and Moshe (1983). For $\lambda\varphi^4$ theory in $(1+1)$-dimensional space see Wang (1991).

2. FREE FIELD IN A TIME-DEPENDENT SPACE-TIME

Four-dimensional de Sitter space is most easily represented as the hyperboloid (Birell and Davies, 1982)

$$Z_0^2 - Z_1^2 - Z_2^2 - Z_3^2 - Z_4^2 = -\alpha^2 \tag{1}$$

embedded in five-dimensional Minkowski space with metric

$$ds^2 = dZ_0^2 - dZ_1^2 - dZ_2^2 - dZ_3^2 - dZ_4^2 \tag{2}$$

The most useful coordinates are those which lead to the line element

$$ds^2 = dt^2 - e^{2t/\alpha} \sum_{i=1}^{3} (dx^i)^2 \tag{3}$$

In terms of the conformal time

$$\eta = -\alpha e^{-t/\alpha}, \qquad -\infty < \eta < 0 \tag{4}$$

the line element in (3) becomes

$$ds^2 = \frac{\alpha^2}{\eta^2}\left[d\eta^2 - \sum_{i=1}^{3} (dx^i)^2\right] \tag{5}$$

thus revealing that this portion of de Sitter space is conformal to half of the Minkowski space. Allowing η to range over all real numbers $-\infty < \eta < \infty$ covers the other half.

The Ricci scalar for the de Sitter space is calculated to be

$$R = -\frac{12}{\alpha^2} \tag{6}$$

which is a constant.

The mode decomposition for the free scalar Klein–Gordon field φ is

$$\varphi(x) = \int d^3k \, [a_k u_k(x) + a_k^+ u_k^*(x)] \tag{7}$$

where $\varphi(x)$ satisfies the Klein–Gordon equation

$$\left[\frac{\eta^2}{\alpha^2}\frac{\partial^2}{\partial\eta^2} - \frac{2\eta}{\alpha^2}\frac{\partial}{\partial\eta} - \frac{\eta^2}{\alpha^2}\Delta + m^2 + \xi R(\eta)\right]\varphi(x) = 0 \tag{8}$$

The modes u_k can be written in a separated form as

$$u_k(x) = (2\pi)^{-3/2}\, e^{ikx}\, \frac{\eta}{\alpha}\, \chi_k(\eta) \tag{9}$$

It is easily seen that $\chi_k(\eta)$ is given by

$$\chi_k(\eta) = \tfrac{1}{2}(\pi\eta)^{1/2} H_\nu^2(k\eta)\, e^{-i(\nu+1/2)\pi} \tag{10}$$

where $H_\nu^{(2)}(k\eta)$ is the second Hankel function and

$$\nu^2 = \tfrac{9}{4} - \alpha^2 m^2 - 12\xi \tag{11}$$

3. $\lambda\varphi^4$ THEORY IN THE SCHRÖDINGER PICTURE

In the Schrödinger-picture field theory (Pi and Samiullah, 1987; Kim *et al.*, 1988) a quantum mechanical wave function $\psi(x, t)$ is replaced by a wave functional $\Psi(\varphi, t)$ which is a functional of a c-number field $\varphi(x)$ at a fixed time t. We take our trial wave functional to be Gaussian, centered at φ and of width G:

$$\begin{aligned}\Psi(\varphi, t) = N \exp\Bigg[&- \int_{x,y} \bar\varphi(x) B(x, y) \bar\varphi(y) + \frac{i}{\hbar}\int_x \hat\pi(x)\bar\varphi(x) \\ &- i \int_{x,y} G(x, y)\Sigma(y, x)\Bigg]\end{aligned} \tag{12}$$

where N is a normalization constant and

$$\begin{aligned}\bar\varphi(x) &\equiv \varphi(x) - \hat\phi(x, t) \\ B(x, y) &\equiv \frac{1}{4\hbar} G^{-1}(x, y, t) - \frac{i}{\hbar}\Sigma(x, y, t)\end{aligned} \tag{13}$$

The expectation values are easily calculated to be

$$
\begin{aligned}
\langle \varphi(x) \rangle &= \hat{\varphi}(x, t) \\
\left\langle -i\hbar \frac{\delta}{\delta_c \varphi(x)} \right\rangle &= \hat{\pi}(x, t) \\
\langle \varphi(x)\varphi(y) \rangle &= \hat{\varphi}(x, t)\hat{\varphi}(y, t) + \hbar G(x, y, t) \\
\left\langle i\hbar \frac{\partial}{\partial t} \right\rangle &= \int_x \hat{\pi}(x, t)\dot{\hat{\varphi}}(x, t) + \hbar \int_{x,y} \Sigma(x, y, t)\dot{G}(y, x, t)
\end{aligned}
\tag{14}
$$

In deriving (14), we have used functional integration, which is slightly modified from the flat-space case as

$$\int D\varphi(x) \to \int D\varphi(x)\, g(x)^{1/4} \tag{15}$$

Also, we have used

$$
\begin{aligned}
\int_x &\equiv \int d^3x \sqrt{g} \\
\delta_c(x, y) &\equiv \frac{1}{\sqrt{g}} \delta(x-y) \\
\frac{\delta \varphi(x)}{\delta_c \varphi(y)} &\equiv \frac{1}{\sqrt{g}} \frac{\delta \varphi(x)}{\delta \varphi(y)} \equiv \delta_c(x, y)
\end{aligned}
\tag{16}
$$

where x, y denote three-dimensional space vectors and g is the determinant of the spatial metric tensor.

The effective action in this picture is given by (Kim *et al.*, 1988)

$$\Gamma = \int dt\, \langle \Psi(t) | i\hbar\partial_t - H | \Psi(t) \rangle \tag{17}$$

The Hamiltonian for a scalar field in curved space is

$$H = \int_x \left[\frac{1}{2}\left(i \frac{\delta}{\delta_c \varphi} \right)^2 + \tfrac{1}{2}(\partial_i\varphi)(\partial_j\varphi)g^{ij} + V(\varphi) \right] \tag{18}$$

where g_{ij} is the spatial metric tensor and g^{ij} is its inverse. [Note that if we keep the metric in the form (3), the metric tensor appears only in the spatial terms. This is important, as the calculations would be very involved otherwise. After evaluating the effective action Γ, we resort to the coordinate change given in (4).]

In the Gaussian approximation, the effective action reduces to

$$\begin{aligned}\Gamma=\int dt\Biggl\{&\int_x [\hat{\pi}\dot{\hat{\varphi}}-\tfrac{1}{2}(\hat{\pi})^2-\tfrac{1}{2}g^{ij}(\partial_i\hat{\varphi})(\partial_j\hat{\varphi})-V(\hat{\varphi})]\\&+\hbar\Biggl[\int_{x,y}\Sigma\dot{G}-2\int_{x,y,z}\Sigma G\Sigma-\int_x (\tfrac{1}{8}\bar{G}^1+\tfrac{1}{2}g^{ij}\,\partial_i^x\partial_j^y G(x,y)|_{x=y}\\&+\tfrac{1}{2}V^{(2)}(\hat{\varphi})G(x,x))\Biggr]\\&-\tfrac{1}{8}\hbar^2\int_x V^{(4)}(\hat{\varphi})G(x,x)G(x,x)\Biggr\}\end{aligned} \tag{19}$$

$\hat{\pi}$, $\hat{\varphi}$, G, and Σ are the variational parameters. $\hat{\pi}$ and Σ play the role of conjugate momenta of $\hat{\varphi}$ and G, respectively.

Differentiating Γ with respect to G and Σ, we get

$$\begin{aligned}&\frac{\delta\Gamma}{\delta G(x,y,t)}=0\\&\quad\to\dot{\Sigma}(x,y,t)+2\int_z \Sigma(x,z,t)\Sigma(z,y,t)\\&\quad=\tfrac{1}{8}\bar{G}^2(x,y,t)-\tfrac{1}{2}[g^{ij}\,\partial_i^x\partial_j^y+V^{(2)}(\hat{\varphi})\\&\qquad+\tfrac{1}{2}\hbar V^{(4)}(\hat{\varphi})G(x,x)]\delta_c(x,y)\end{aligned} \tag{20}$$

and

$$\begin{aligned}&\frac{\delta\Gamma}{\delta\Sigma(x,y,t)}=0\\&\quad\to\dot{G}(x,y,t)\\&\quad=2\int_z [G(x,z,t)\Sigma(z,y,t)+\Sigma(x,z,t)G(z,y,t)]\end{aligned} \tag{21}$$

Using equations (3) and (4), we find that, in our case, g^{ij} can be written in terms of η as

$$g^{ij}=\frac{\eta^2}{a^2} \tag{22}$$

Now we introduce an effective mass term (Kim *et al.*, 1988)

$$\Omega^2 \equiv V^{(2)}(\hat{\varphi}) + \frac{\hbar}{2} V^{(4)}(\hat{\varphi}) G(x, x) \tag{23}$$

Using equations (22) and (23), we obtain from (20) and (21) and the definition of η,

$$\frac{1}{2}\frac{\eta^2}{\alpha^2}\left\{\frac{\partial^2}{\partial \eta^2} G(x, y, \eta) + \frac{1}{\eta}\frac{\partial}{\partial \eta} G(x, y, \eta) - \tfrac{1}{2}\bar{G}^1(x, y, \eta)\left[\frac{\partial}{\partial \eta} G(x, y, \eta)\right]^2\right\}$$
$$= \tfrac{1}{4}\bar{G}^2(x, y, \eta) - \left(\frac{k^2\eta^2}{\alpha^2} + \Omega^2\right)\delta_c(x, y) \tag{24}$$

In analogy with the static flat space, we can consider a static case where G and $\bar{G}^1$ are independent of η. Equation (24) then reduces to

$$\tfrac{1}{4}\bar{G}^1(x, y) = \left(\frac{\eta^2 k^2}{\alpha^2} + \Omega^2\right) G(x, y)\, \delta_c(x, y) \tag{25}$$

The effective action then reduces to an effective potential given by

$$\Gamma = -\int d^4x\, V_{\text{eff}} \tag{26}$$

where

$$\begin{aligned} V_{\text{eff}} = {} & V(\hat{\varphi}) + \frac{\hbar}{2} V^{(2)}(\hat{\varphi}) G(x, x) \\ & + \frac{\hbar^2}{8} V^{(4)}(\hat{\varphi})[G(x, x)]^2 + \frac{\hbar}{8}\bar{G}^1(x, x) \\ & + \frac{1}{2}\frac{\eta^2}{\alpha^2}[\partial_i\hat{\varphi}(x)\, \partial_j\hat{\varphi}(y) + \hbar\, \partial_i^x\partial_j^y G(x, y)]|_{x=y} \end{aligned} \tag{27}$$

If we consider $\hat{\varphi}$ to be a constant, equation (27) reduces to

$$V_{\text{eff}} = V(\hat{\varphi}) + \frac{\hbar}{4}\bar{G}^1(x, x) - \frac{\hbar^2}{8} V^{(4)}(\hat{\varphi})[G(x, x)]^2 \tag{28}$$

In deriving (28) from (27), we have used equations (23) and (25).

Confining ourselves to a de Sitter space-time with potential

$$V(\hat{\varphi}) = \tfrac{1}{2}(m^2 + \xi R)\hat{\varphi}^2 + \lambda\hat{\varphi}^4 \tag{29}$$

our effective potential takes the form

$$V_{\text{eff}} = \tfrac{1}{2}(m^2 + \xi R)\hat{\varphi}^2 + \lambda\hat{\varphi}^4 + \frac{\hbar}{4}\bar{G}^1(x, x) - 3\lambda\hbar^2[G(x, x)]^2 \tag{30}$$

Henceforth we shall put $\hbar = 1$.

4. EVALUATION OF G AND $\bar{G}^1$

G and $\bar{G}^1$ are the solutions of equation (25),

$$\tfrac{1}{4}\bar{G}^1(x, y) = \left(\frac{\eta^2 k^2}{\alpha^2} + \Omega^2\right) G(x, y)\ \delta_c(x, y)$$

Sandwiching equation (25) between the normalized eigenfunctions $u_k(x)$ of the free field of de Sitter space, given in equation (9) above, we obtain

$$\begin{aligned}\frac{\bar{G}^1}{4}(x, x)\ \frac{1}{8\pi\alpha^2}\int_0^\infty d\rho\ \rho^2 |H_\nu^{(2)}(\rho)|^2\ e^{\pi\ \mathrm{Im}\ \nu}\\ = G(x, x)\ \frac{1}{8\pi\alpha^4}\int_0^\infty d\rho\ \rho^2(\rho^2 + \alpha^2\Omega^2)|H_\nu^{(2)}(\rho)|^2\ e^{\pi\ \mathrm{Im}\ \nu}\end{aligned} \tag{31}$$

where $\rho \equiv k\eta$.

Since $G(x, x) = \langle\bar{\varphi}(x)\bar{\varphi}(y)\rangle$, the solutions of (31) are

$$G(\Omega) = \frac{1}{8\pi\alpha^2}\int_0^\infty d\rho\ \rho^2 |H_\nu^{(2)}(\rho)|^2\ e^{\pi\ \mathrm{Im}\ \nu} \tag{32}$$

and

$$\tfrac{1}{4}\bar{G}^1(\Omega) = \frac{1}{8\pi\alpha^4}\int_0^\infty d\rho\ \rho^2(\rho^2 + \alpha^2\Omega^2)|H_\nu^{(2)}(\rho)|^2\ e^{\pi\ \mathrm{Im}\ \nu} \tag{33}$$

where

$$\nu^2 = \tfrac{9}{4} - \alpha^2\Omega^2 \tag{34}$$

In deriving the above, we have redefined m^2 and Ω^2 as

$$\begin{aligned} m^2 &\equiv m^2 + 6\xi/\alpha^2 \\ \Omega^2 &\equiv \Omega^2 + 12\xi/\alpha^2 \end{aligned} \tag{35}$$

This asymmetry in the definition of the mass terms is employed to cancel the curvature-dependent divergences in the final expression for the theory.

5. RENORMALIZATION OF THE EFFECTIVE POTENTIAL

The effective potential in (30) takes the form

$$V_{\text{eff}} = \tfrac{1}{4}G^{-1}(\Omega) + \tfrac{1}{2}(m^2 - \Omega^2)G(\Omega) + \tfrac{1}{2}m^2\hat{\varphi}^2 + \lambda\hat{\varphi}^4 + 6\lambda\hat{\varphi}^2 G(\Omega) - 3\lambda[G(\Omega)]^2 \tag{36}$$

which is in agreement with that of Latorre *et al.* (1986) if we make the following identifications:

$$\begin{aligned} I_0(\Omega) &\equiv G(\Omega) \\ I_1(\Omega) &= \tfrac{1}{4}\bar{G}^1(\Omega) \end{aligned} \tag{37}$$

The expression for V_{eff} contains many divergences which should be removed by renormalization of the mass and the coupling constant λ, and by the subtraction of the zero-point energy.

First of all, we evaluate G and $\bar{G}^1$ from equations (32) and (33). Writing

$$H_\nu^{(2)}(\rho) = i\,\text{cosec}(\nu\pi)\,[J_{-\nu}(\rho) - e^{\nu\pi i}J_\nu(\rho)] \tag{38}$$

we obtain with the help of ε regularization, after some straightforward calculations (with $\varepsilon \to 0$),

$$G(\Omega) = \frac{1}{8\pi^2\alpha^2}\left[S_0(\Omega) + \frac{1}{\varepsilon}(\alpha^2\Omega^2 - 2)\right] \tag{39}$$

$$\tfrac{1}{4}\bar{G}^1(\Omega) = \frac{1}{8\pi^2\alpha^4}\left[S_1(\Omega) + \frac{1}{4\varepsilon}A^2\Omega^2(\alpha^2\Omega^2 - 2)\right] \tag{40}$$

where

$$S_0(\Omega) = \tfrac{1}{2}(\tfrac{1}{4} - \nu^2)[3 - 2\psi(2) + \psi(\tfrac{3}{2} - \nu) + \psi(\tfrac{3}{2} + \nu) - \ln 4] \tag{41}$$

$$\begin{aligned} S_1(\Omega) = {} & \tfrac{1}{2}(\tfrac{1}{4} - \nu^2)\alpha^2\Omega^2[3 - 2\psi(2) + \psi(\tfrac{3}{2} - \nu) + \psi(\tfrac{3}{2} + \nu) - \ln 4] \\ & + \tfrac{1}{16}(\tfrac{1}{4} - \nu^2)(\tfrac{9}{4} - \nu^2)[-25 + 12\psi(3) \\ & - 6\psi(\tfrac{5}{2} - \nu) - 6\psi(\tfrac{5}{2} + \nu) + 6\ln 4] \end{aligned} \tag{42}$$

and

$$S_1(\Omega) = \tfrac{1}{4}\alpha^2\Omega^2 S_0(\Omega) + \tfrac{9}{4} - \alpha^2\Omega^2 - \tfrac{1}{16}\alpha^4\Omega^4 \tag{43}$$

Equations (39) and (40) are derived in the following way. In calculating G and $\bar{G}^1$, we encounter integrals of the following type (Gradshteyn and Ryzhik, 1980):

$$\int_0^\infty J_\nu(t)J_\mu(t)t^{-\lambda}\,dt$$

$$=\frac{\Gamma(\lambda)\Gamma((\nu+\mu-\lambda+1)/2)}{2^\lambda\Gamma((-\nu+\mu+\lambda+1)/2)\Gamma((\nu+\mu+\lambda+1)/2)\Gamma((\nu-\mu+\lambda+1)/2)} \tag{44}$$

In our case, λ is a negative integer. To make the integrals finite, we replace λ by $\lambda+\varepsilon$ with $\varepsilon\to 0$. Also, we make use of the following properties of gamma and digamma functions

$$\begin{aligned}\Gamma(a+\varepsilon)&\simeq\Gamma(a)[1+\varepsilon\psi(a)]+o(\varepsilon^2)\\ \Gamma(a)\Gamma(1-a)&=\pi\operatorname{cosec}(a\pi)\end{aligned} \tag{45}$$

$$\begin{aligned}\psi(1-z)&=\psi(z)+\pi\cot(\pi z)\\ \psi(2z)&=\tfrac{1}{2}\psi(z)+\tfrac{1}{2}\psi(z+\tfrac{1}{2})+\ln 2\end{aligned} \tag{46}$$

Using the results (44)–(46) and (38) and the definitions of $G(\Omega)$ and $\bar{G}^1(\Omega)$, we easily obtain the results (39)–(41). Apart from simplicity, this method has the added advantage that, as in dimensional regularization, the divergences appear as poles in ε. Now, minimizing V_{eff} with respect to Ω and $\hat{\varphi}$ yields

$$\left.\frac{\partial V_{\text{eff}}}{\partial\Omega}\right|_{\bar{\Omega}}=0\to\bar{\Omega}^2=m^2+12\lambda\hat{\varphi}_0^2+12\lambda G(\bar{\Omega}) \tag{47}$$

$$\frac{\partial V_{\text{eff}}}{\partial\hat{\varphi}}=0\to\hat{\varphi}_0[m^2+4\lambda\hat{\varphi}_0^2+12\lambda G(\bar{\Omega})]=0 \tag{48}$$

Thus, equation (48) has two solutions for $\hat{\varphi}_0$.

(i) $\hat{\varphi}_0=0$ restores the symmetry.

(ii) $\hat{\varphi}_0^2=-(1/4\lambda)[m^2+12\lambda G(\bar{\Omega})]$ induces spontaneous symmetry breaking.

We discuss here the spontaneously broken phase.

From (47) and (48), we obtain

$$\bar{\Omega}^2=8\lambda\hat{\varphi}_0^2 \tag{49}$$

Since $\bar{\Omega}^2>0$, λ must be positive.

Substituting the expression for $G(\Omega)$ from (39) in equation (47), we get, using (49),

$$-\tfrac{1}{2}\bar{\Omega}^2=\bar{\Omega}_0^2+\frac{3\lambda}{2\pi^2\alpha^2}\left[S_0(\bar{\Omega})-S_0(\bar{\Omega}_0)+\frac{\alpha^2}{\varepsilon}(\bar{\Omega}^2-\bar{\Omega}_0^2)\right] \tag{50}$$

The renormalized mass is defined by

$$
\begin{aligned}
m^2 &= \left.\frac{\partial^2 V_{\mathrm{eff}}}{\partial \hat{\varphi}^2}\right|_{\hat{\varphi}=0} \\
&= m^2 + 12\lambda G(\bar{\Omega}_0) \\
&= \bar{\Omega}_0^2
\end{aligned}
\tag{51}
$$

The renormalized coupling constant is defined by

$$
\lambda_R = \left.\frac{1}{4!}\frac{\partial^4 V_{\mathrm{eff}}}{\partial \hat{\varphi}^4}\right|_{\hat{\varphi}=0} \tag{52}
$$

or

$$
\lambda_R = \left.\frac{\lambda\{[\frac{1}{4}(\bar{G}^1)'' - G']G' + [\frac{1}{2}G - \frac{1}{4}(\bar{G}^1)']G'' - 12\lambda(G')^3\}}{[\frac{1}{4}(\bar{G}^1)'' - G']G' + [\frac{1}{2}G - \frac{1}{4}(\bar{G}^1)']G'' + 6\lambda(G')^3}\right|_{\hat{\varphi}=0} \tag{53}
$$

where

$$
G' = \left.\frac{\partial G}{\partial \Omega^2}\right|_{\bar{\Omega}_0^2}, \qquad (\bar{G}^1)' = \left.\frac{\partial \bar{G}^1}{\partial \Omega^2}\right|_{\bar{\Omega}_0^2}, \qquad \text{etc.}
$$

Substituting equations (39)–(43) in (53), we obtain in the $\varepsilon \to 0$ limit

$$
\lambda_R = -2\lambda \tag{54a}
$$

Thus, $\lambda_R < 0$ if λ is positive. This is analogous and in fact identical to the flat-space result of Consoli and Ciancitto (1985). Since λ and λ_R are connected by a finite relation, the divergences in equation (50) and hence in the expression for V_{eff} do not cancel.

Solving the quadratic equation in λ in equation (53), we obtain the second root as

$$
\lambda = \tfrac{2}{3}\pi^2\varepsilon + O(\varepsilon^2) \tag{54b}
$$

This value of λ cancels the divergence in equation (50), but it fails to give a nonzero extremum of the potential. Therefore, we are compelled to abandon this renormalization for λ.

Next we try an alternative method for renormalizing λ.

Consider equation (50). Scaling $\bar{\Omega}_0^2 = 0$, we obtain

$$
\frac{1}{2}\bar{\Omega}^2\left(1 + \frac{3\lambda}{\pi^2\varepsilon}\right) = \frac{3\lambda}{2\pi^2\alpha^2}[S_0(\bar{\Omega}_0) - S_0(\bar{\Omega})] \tag{55}
$$

To make this last expression finite, we must take the following definition for λ:

$$\lambda = -\frac{\pi^2 \varepsilon}{3} - a\varepsilon^2 \tag{56}$$

Substituting the value of λ from equation (56) in equation (55), we obtain the following expression for a:

$$a = +\frac{\pi^2}{3\alpha^2\bar{\Omega}^2}[S_0(\bar{\Omega}_0) - S_0(\bar{\Omega})] \tag{57}$$

In the flat-space limit, i.e., $\alpha^2\bar{\Omega}^2 \to 2$, equation (57) yields

$$a = +\frac{\pi^2}{6} \tag{58}$$

Now, equation (56) can be written as

$$\frac{1}{\varepsilon} = -\frac{\pi^2}{3\lambda}\left(1 - \frac{9a\lambda}{\pi^4}\right) \tag{59}$$

Therefore, in the absence of curvature, equation (59) can be written as [substituting the value of a from (58)],

$$\frac{1}{\varepsilon} = -\frac{\pi^2}{3\lambda}\left(1 - \frac{3\lambda}{2\pi^2}\right) \tag{60}$$

Comparing with the flat-space result, we find $I_{-1} \sim -3/\pi^2\varepsilon$, which implies (i) $\varepsilon < 0$ and (ii) $\lambda < \frac{2}{3}\pi^2$.

With this identification we get back the result of Consoli and Ciancitto (1985) in the flat-space limit [remember that their λ differs from ours by a numerical factor (24 to be precise)].

Thus, meaningful physics can arise only if λ is positive and infinitesimal (at least when the curvature term is not too large). The subtracted energy density then turns out to be

$$\begin{aligned}
V_{\text{eff}} - D &= \tfrac{1}{4}\bar{G}^1(\bar{\Omega}) - \tfrac{1}{4}\bar{G}^1(\bar{\Omega}_0) + \lambda\varphi_0^4 \\
&\quad - \tfrac{1}{2}\bar{\Omega}^2 G(\bar{\Omega}) + 3\lambda[G(\bar{\Omega}) - G(\bar{\Omega}_0)]^2 \\
&\quad + 6\lambda\hat{\varphi}_0^2[G(\bar{\Omega}) - G(\bar{\Omega}_0)] \\
&= -\frac{\bar{\Omega}^4}{128\pi^2} - \frac{S_0(\bar{\Omega})}{16\pi^2\alpha^4} \\
&\quad + \frac{\bar{\Omega}^2}{32\pi^2\alpha^2}[S_0(\bar{\Omega}) - 2S_0(\bar{\Omega}_0) - 4]
\end{aligned} \tag{61}$$

As can be seen from (61), our result reproduces the flat-space result of Consoli *et al.* (1985) in the absence of the curvature term. Equation (61) shows that the absolute minimum in the Gaussian functional is obtained and hence spontaneous symmetry breaking occurs as in the flat-space case. However, the restriction on λ now depends on the curvature and the upper limit differs considerably from $\frac{2}{3}\pi^2$ when α is a finite. To be precise, $\lambda \leq \pi^4 9a$, where a is given by equation (57).

6. DISCUSSIONS AND CONCLUSIONS

In this paper we discuss spontaneous symmetry breaking of $\lambda\varphi^4$ field theory in a time-dependent space-time using the Schrödinger picture. So far as we know, this is the first attempt to deal with time-dependent space-time in the Schrödinger picture. It is seen that the approach yields the effective potential in an elegant way. Moreover, our normalization procedure is different from that used by previous authors for $\lambda\varphi^4$ theory in curved space. Using the normalization procedure of Consoli *et al.*, we have been able to show that $\lambda\varphi^4$ theory undergoes spontaneous symmetry breaking. In this normalization scheme Ω_0 has been chosen to be zero. To have a meaningful theory, λ must have an upper limit which depends on the curvature term α. Our formulation yields all the results of the flat-space case when the proper limit is taken. An interesting extension of this work would be in the realm of finite temperature to see if there exists a critical temperature T_c above which symmetry is restored. T_c will obviously depend on the curvature. Work is in progress along this line.

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